CAS and the first groups in mathematics, 1993-95.

Olav Arnfinn Laudal, Professor, University of Oslo

Norway has over the years fostered a long line of great mathematicians, very few of whom are well known in their home country. Most Norwegians will, however, know about the two Giants from the 19th century, Niels Henrik Abel and Sophus Lie, but names like Sylow, Thue, Heegaard, Brun, Skolem, Ore, Nagel, Selberg and Selmer, held in great esteem in international mathematical circles, will most certainly go unnoticed outside the small group of professional scientists, in Norway.

Recently the mathematical community, aligning all four Universities and with support from many of the different Colleges that were established at the end of the 20th century, was determined to do something about this situation. Of course, the fact that few Norwegians know that Sylow is a name that all serious mathematicians, physicists, astrophysicists, chemists, all those who have to know something about symmetry and therefore about groups, in this case finite groups, have to learn, is not in itself so shameful. What concerns us is that this lack of biographical facts shows that mathematical science as such is considered a closed book to most people today, and one that many, even culturally keenly interested people, would like to see lost.

This situation has its historical reasons, not so different in this nation from what we see in most industrial societies today. Mathematics has lost its visibility, everywhere, and the sciences that are still seen by people, are those that harbour personalities with special relations to the media. And, of course, Big Science, the industrially interesting immediately applicable science, is publicised everywhere.

One of the most effective means to make the wonders of mathematics shine on the Norwegians is therefore to use the two lighthouses, Abel and Lie, and keep them activated.

This was the purpose of the Sophus Lie Memorial Week 1992, celebrating the 150 year anniversary of Lie, and this is the purpose of the Niels Henrik Abel Bicentennial, and its Conference in Oslo June 3-8, this year. Both arrangements were beautifully seconded by the monumental biographies of Abel and Lie written by Arild Stubhaug.

During the preparations for the Sophus Lie anniversary in 1992, an old idea came up again, that Norway deserved a Mathematical Research Centre, an Abel-Lie Institute. Denmark has its world famous Niels Bohr Institute, Finland has its Nevanlinna Institute, and Sweden has its Mittag Leffler Institute, Norway had nothing like it.

Then, in the rapidly decaying ruins of the Soviet Empire, there were people trying to survive, struggling to continue the work of the great mathematical schools of Moscow, Leningrad, Novosibirsk, Kiev, Minsk etc. Some of them got together and established an International Sophus Lie Centre, ISLC, in Minsk, with the purpose of promoting doctoral training, and research in differential equations along the lines proposed by Sophus Lie in the 1880s, but later abandoned, because of the complexity of the necessary computations. The new computer technology turned out to be an efficient tool to progress in the direction of research, opened up by Sophus Lie.

Professor Komrakov, one of the founders of the ISLC, contacted the organising committee of the Sophus Lie Memorial Week, and the Department of Mathematics at the University of Oslo

decided to apply for money allocated by the Government for supporting education and research in the defunct Soviet Union, to support part of the activity of the ISLC.

The Department got a share of this money, and when, some time in the Fall of 1992, the then President of the Board of the CAS, Professor Vigdis Ystad, proposed that I organise a year in mathematics at the CAS, starting in the Fall of 1993, some of the funds needed for this project were already available. But, in contrast to the situation today, where the CAS has a budget providing the research groups with a fairly good economy, the situation during the two years, 1993-1995, when I organised the Mathematics group, required a constant uphill battle against the Norwegian Research Council, the universities, and the ministries concerned.

My own application for a partial Sabbatical during these years, was turned down, by the Department of Mathematics, due to lack of support from the University of Oslo.

Finally, with a generous economic guaranty provided by the University of Oslo, of which the University lost a large part, the mathematics program could be pulled through. During the four semesters 1993-1995, the CAS was host to 49 mathematicians from 13 different countries, working on Lie theory, Algebraic Geometry, Analytic Manifolds, Singularity theory, Differential Geometry, K-theory, Operator Algebra, Group representations, Applied Mathematics, and Theoretical Physics.

The guests wrote 38 papers while at the CAS, several book projects got under way, or were completed, and the invited guests gave, together, more than 50 lectures, at the CAS or at the Department of Mathematics, in Oslo, Bergen, Trondheim or Tromsø.

As I have hinted at above, it is very difficult to give a serious account of what the subjects studied at the CAS during these two years, were really about, at least while keeping the interest of a general audience. Let me just choose one example. Most people know that there are problems related to solving polynomial equations in one variable. Niels Henrik Abel was the first to prove that the equations of degree 5 or higher, in general, have no solution that can be expressed as a radical function in the coefficients, i.e. as a function combining only addition/subtraction, multiplication/division and root-extraction. He also introduced what later became the Abelian part of Galois theory, a *symmetry group* associated to any polynomial, from which one can read out when the polynomial equation has solutions which are radical functions of the coefficients.

Sophus Lie knew the result achieved by Abel, and attended Sylow's lectures on Galois theory at the end of his studies in Oslo, then Christiania. He understood some years later that there is an analogous *symmetry group* for any system of differential equations, from which one can read out properties of the solutions of the system. These groups have since been named after Lie, and there is today no serious University in the world that does not offer a course in Lie theory, teaching mathematicians and physicists and chemists about the inner workings of Lie groups.

As related above, the study of these symmetry groups for various systems of differential equations was the main idea behind the teaching and research offered at the ISLC, and the co-operation at the CAS between Norwegian mathematicians and the guests linked with the ISLC brought new insights into Sophus Lie's integration method back to his homeland.

The guests were unanimous in their praise of the CAS, the working conditions, and the services rendered by the then administrator, Mrs. Unn Hagen.

I was a little surprised by the low level of criticism, since I still think that the lack of a scientific library is a negative feature of the CAS. Compared to the research conditions at the Mittag Leffler Institute, or at the German Oberwolfach Institute, with their magnificent mathematical libraries, the CAS has little to offer a working mathematician. Nevertheless experience has shown that the CAS functions as a secluded workshop and as a meeting place for scientists and researchers having to cope, at least during lunch breaks, with people with completely different agendas. In 1993-1994 mathematicians and Ibsen researchers lived together and became friends.

This year, 2001-2002, mathematics has come back to the CAS. The subject is this time Non-commutative Geometry, including Operator theory, Representation theory, and Algebraic Geometry. I am sure the final report of this group will be as positive as the one for the starting years 1993-1995.

Now, can one draw some further conclusions from the experience, so far, of the CAS as an institution, apart from the interpersonal ones expressed above?

Seen from the point of view of mathematics, a science that, to a great degree, is still independent of technical equipment, the meeting place is our laboratory.
That is why the workshops abound these days, financed by a plethora of international and European programs and projects. It is obviously a common good to meet and learn. But there is also an obvious need for longer periods of independent work, and for collaboration between two or a few researchers, to cook up new ideas.

New ideas are very rare indeed, and their gestation may be very long and agonising. A university professor, with the daily duties of teaching and administration, not to mention the almost daily paperwork needed for reporting and fund-raising, has today little chance to carry it through to birth, even when he or she is clearly pregnant with a new idea.

Here is where the research centres like the CAS come in. They provide an essential escape from the daily routines, a haven for thinking.

On the national level, the net result that I can see clearly, stemming from the activities of the mathematics groups at the CAS also includes, together with a lot of mathematical co-operation between Norwegian mathematicians and former guests at CAS, at least three mathematical imports, resulting from the appointment of Valentin Lychagin, a prominent guest during 1993-95, as a professor at The University of Tromsø.

In the future I would hope that the CAS can stay economically alive and sound, and in the spirit of the Bjørgo Report, § 11, evaluating the first years of the CAS, function as this haven for thinking, and as a much needed meeting place, providing contacts between international and Norwegian research, as we wait for an independent Abel-Lie Mathematical Research Centre.